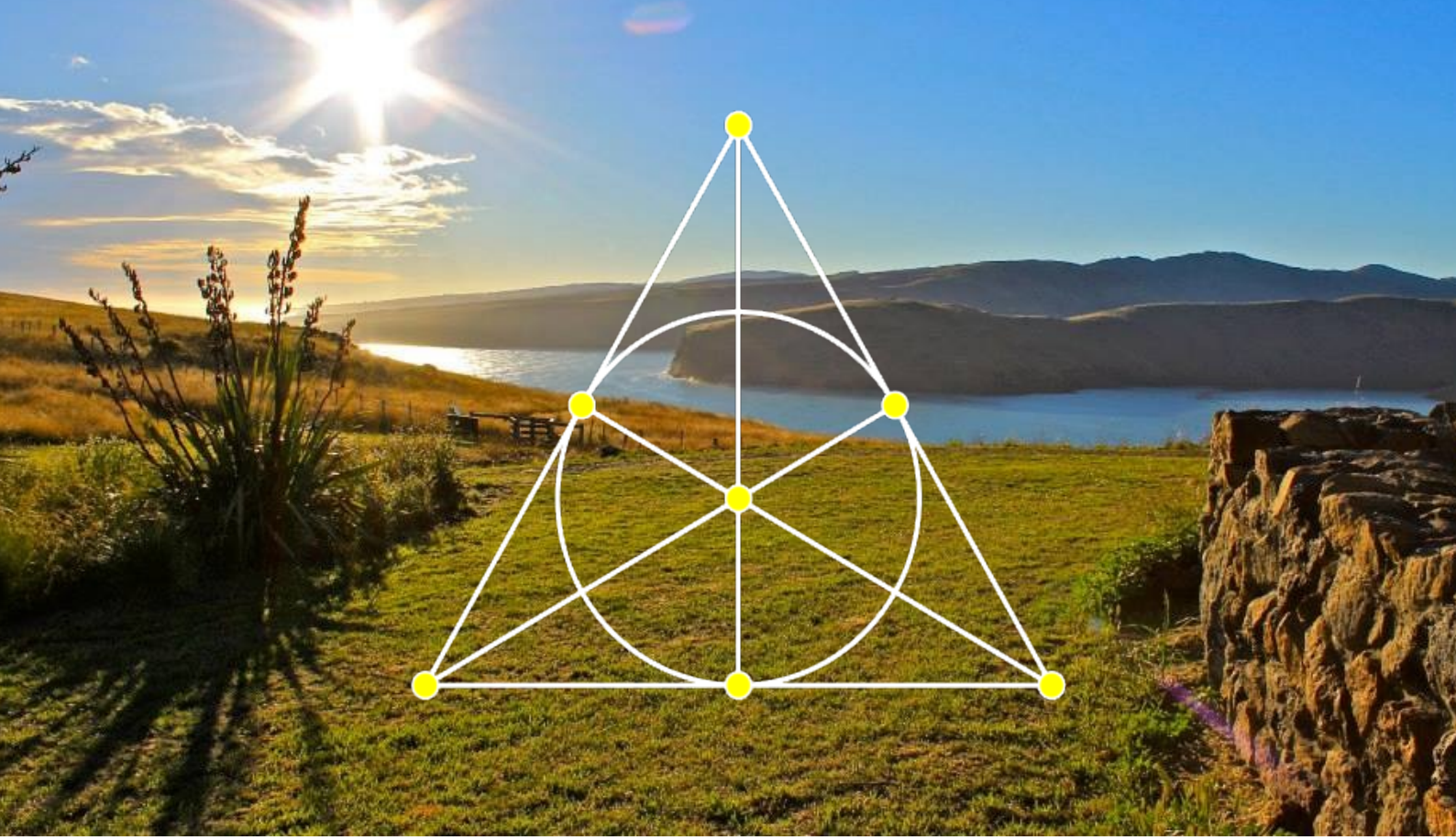


Combinatorics in Christchurch

University of Canterbury
4 - 6 June, 2024

Organisers

Jesse Lansdown
University of Canterbury

Geertrui Van de Voorde
University of Canterbury

Sponsors

Keynote Speaker

Bill Martin
Worcester Polytechnic Institute, USA

Invited Speakers

Carmen Amarra
University of the Philippines Diliman, Philippines

John Bamberg
The University of Western Australia, Australia

Gary Greaves
Nanyang Technological University, Singapore

Jeroen Schillewaert
University of Auckland, New Zealand

Anita Liebenau
University of New South Wales, Australia

Sho Suda
National Defense Academy of Japan, Japan

The Institute of Combinatorics and its Applications

UC UNIVERSITY OF CANTERBURY
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

Contents

Welcome!

Kia ora koutou!

Our goal for *Combinatorics in Christchurch* is to bolster the combinatorial community in New Zealand by bringing together mathematicians working in combinatorics and related fields. We have been encouraged by the large number of registrations, participation by students, and the calibre of our speakers. We hope that this conference will be a fruitful time of sharing research, exchanging ideas, and beginning future collaborations.

We are particularly grateful for financial support from the Faculty of Engineering at the University of Canterbury, the Institute of Combinatorics and its Applications, and the New Zealand Mathematical Society. We also thank the the administrative staff of the School of Mathematics and Statistics for their help. Finally, we are grateful for the UC Erskine Fellows program for bringing Bill Martin to Christchurch and providing the impetus for this conference.

Information regarding the conference (including a copy of this booklet) can be found at the conference website: https://combinatoricsinchristchurch.github.io/. Please check there regularly for any updates or changes.

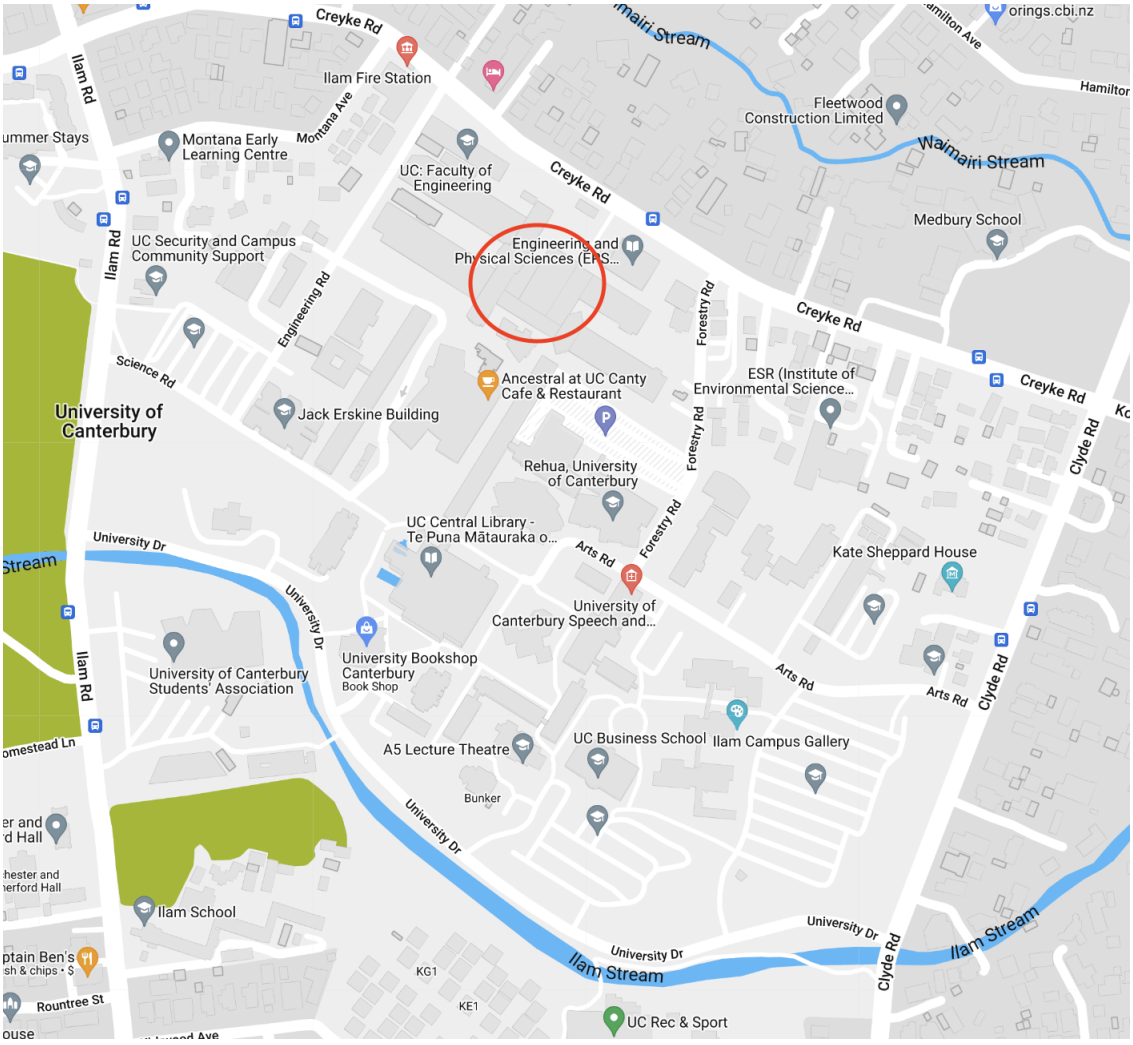
We wish you a pleasant time in Christchurch!

The organisers:
Jesse Lansdown
Geertrui Van de Voorde

The cover image is of Godley Head in Christchurch and is kindly supplied by Yo Ruiters of the UC School of Mathematics and Statistics.

Map

All talks will be in Rātā/Engineering Core E16 Lecture Theatre



On google maps: https://maps.app.goo.gl/cZQAA6n4ic58mEcq9

Schedule

All talks will be in Rātā – Engineering Core E16 Lecture Theatre

	Tuesday	**Wednesday**	**Thursday**
8:30 - 8:50	Registration		
8:50 - 9:00	Welcome		
9:00 - 9:30	**Greaves** 11	**Schillewaert** 11	**Bamberg** 10
9:30 - 10:00	**Greaves cont.**	**Schillewaert cont.**	**Bamberg cont.**
10:00 - 10:30	Morning tea	Morning tea	Morning tea
10:30 - 11:00	Basit 13	Petit 16	Linz 15
11:00 - 11:30	Bastida 13	Steinke 17	McLachlan 15
11:30 - 12:00	**Suda** 12	Searles 16	Croucher 14
12:00 - 12:30	**Suda cont.**	Slattery-Holmes 17	Nenadov 16
12:30 - 13:00	Photo + Lunch	Lunch	Lunch
13:00 - 13:30	Lunch	Lunch	Lunch
13:30 - 14:00	Seong 17	**Martin** 9	**Liebenau** 11
14:00 - 14:30	Young 18	**Martin cont.**	**Liebenau cont.**
14:30 - 15:00	Conder 14	**Martin cont.**	
15:00 - 15:30	Lehner 14		
15:30 - 16:00	Afternoon tea		
16:00 - 16:30	**Amarra** 10		
16:30 - 17:00	**Amarra cont.**		
		Dinner*	

*self-funded, see notes below.

Notes:

- There is no official conference dinner, but we have reserved some tables at **Little High Eatery** (https://www.littlehigh.co.nz/), 255 St. Asaph street, on Wednesday 5 June, 6pm.
- Keynote, invited, and contributed talks are 90 minutes, 50 minutes, 25 minutes, respectively. This includes time for questions.
- Wi-Fi is available via Eduroam using credentials from your home institution.
- See "Local Information" on the website for coffee and food suggestions.

Keynote

Lines with two angles

William J. Martin

Worcester Polytechnic Institute, USA

The ten diagonals of a regular dodecahedron in $\mathbb{R}^3$ give us a "projective double" of the Petersen graph P in the sense that we have ten lines through the origin in bijective correspondence with the vertices of P with the property that the angle between two lines depends only on whether the corresponding vertices of P are adjacent. One can easily contruct a projective double of a graph Γ having m edges in $\mathbb{R}^m$, but what is the smallest dimension for which we can do this, given Γ? Must projective doubles in small dimensions have some extra regularity?

The projective doubles in the talk arise naturally from very regular combinatorial structures called *association schemes*. We focus on the case where one of the two angles is $90°$. We will explore numerous examples. One family of examples are called "mutually unbiased bases". These are the real versions of complex sets of lines describing "optimally independent" measurements of a finite-dimensional quantum system. Several open questions about these will be discussed. Another family of examples, discovered by Penttila and Williford, come from doubly subtended subquadrangles of generalised quadrangles $GQ(q, q^2)$.

The second half of the talk will delve into non-existence results using the theory of association schemes. We will introduce these as finite subsets of the unit sphere in $\mathbb{R}^m$ whose pairwise inner products interact in a nice way with the ring of polynomials in m real variables. In this talk, we are interested in lines through the origin; the corresponding association schemes are called *Q-bipartite*. Viewed as spherical codes (where each line intersects the sphere in two places), we have $d = 4$ non-zero angles in this case.

The talk is based in part on joint work with Brian Kodalen and joint work with Jason Williford and the work is supported by the US National Science Foundation.

Invited talks

Block-transitive, point-imprimitive 2-designs

Carmen Amarra

University of the Philippines Diliman, Philippines

A 2-(v, k, λ) block design is an incidence structure consisting of points and blocks, such that v is the total number of points, k is the number of points in each block, and any two distinct points lie in exactly λ blocks. An automorphism of a design is a permutation of points that leaves the block set invariant, and a design is block- transitive if it admits an automorphism group that has one orbit on the block set. In 1989 Delandtsheer and Doyen showed that for a fixed v and k, almost all block-transitive t-(v, k, λ) designs are point-primitive, that is, the automorphism group has no nontrivial invariant partition on the point set. Block-transitive, point-primitive 2-designs are well-studied; less is known of the point-imprimitive case. In this talk we will consider block-transitive 2-designs which have two or more imprimitive partitions. I will discuss recent work with Alice Devillers, Cheryl Praeger, Seyed Hassan Alavi, and Ashraf Daneshkhah, on the "chain" and "grid" cases (which describe the relationship among the imprimitive partitions), and in particular describe our constructions of infinite families of such designs.

Two-character caps of finite projective spaces

John Bamberg

The University of Western Australia, Australia

This talk is on the connections between uniformly packed 1-error correcting linear codes, partial quadrangles, and two-character caps of finite projective spaces. We first give an overview of the objects at play, and the connections between them, before giving a (beautiful) application of design-orthogonality of subsets in an association scheme to an open problem on two-character caps. No knowledge of finite geometry will be assumed, and effort with be expended on making the topic as approachable as possible to a general audience.

Edge-regular graphs with regular cliques

Gary Greaves

Nanyang Technological University, Singapore

A graph Γ is called **edge-regular** if each of its edges is contained in the same number of triangles of Γ. A clique C in a graph Γ is called **regular** if each vertex outside C is adjacent to the same number of vertices in C. In 1981, Arnold Neumaier showed that every vertex-transitive and edge-transitive *edge-regular* graph that has a *regular clique* must be strongly regular. This motivated him to ask if there exist any edge-regular graphs with regular cliques that are *not* strongly regular. In 2018, Greaves and Koolen answered Neumaier's question by providing an infinite family of edge-regular graphs that have regular cliques but are not strongly regular. Since 2018, there has been a surge of interest from the community, which has led to various new constructions of such graphs (now referred to as Neumaier graphs). I will give an overview of the topic of Neumaier graphs, present some constructions, and provide new a perspective of their study via the coherent closure of a graph.

Ramsey with purple edges

Anita Liebenau

University of New South Wales, Australia

Motivated by a question of David Angell, we study a variant of Ramsey numbers where some edges are coloured both red and blue, or purple. Specifically, we are interested in the largest number $g = g(s, t, n)$, for some s and t and $n < R(s, t)$, such that there exists a red-blue-purple colouring of K_n with g purple edges, without a red-purple K_s and without a blue-purple K_t. We determine g asymptotically for a large family of parameters. The talk will be introductory in nature. Since the concept of double-coloured, or purple, edges is new in this context, there is a plethora of open questions. Joint work with Thomas Lesgourgues and Nye Taylor.

Geometric and combinatorial group theory

Jeroen Schillewaert

University of Auckland, New Zealand

Klein's combination theorem (the ping-pong lemma) is a combinatorial method to detect free subgroups in a given group. More elaborate versions are used to answer the central problem of this talk: Classify discrete groups generated by two matrices in $\mathrm{SL}(2, K)$, where K is a local field.

I will briefly discuss the real case, which was settled in the eighties, and some results in the wide-open complex case (with Alex Elzenaar and Gaven Martin).

The focus of this talk will be on the remaining (non-Archimedean) case which Matthew Conder and I recently solved combining the study of group actions on trees with basic number theory.

Uniqueness of an Association Scheme Obtained from Hoggar's 64 Lines in $\mathbb{C}^8$

Sho Suda

National Defense Academy of Japan, Japan

In 1978 and 1982, Hoggar constructed a set of unit vectors in $\mathbb{C}^8$ with absolute values of inner products equal to $\frac{1}{3}$. This set forms equiangular lines and is also known as SIC-POVMs. In 2014, Roy and I verified that the fourfold cover of this set forms a commutative association scheme on 256 vertices with 7 binary relations determined by the inner products.

In this talk, we show that this commutative association scheme is unique up to isomorphism by its parameters. Our tools for the proof are triple intersection numbers and spherical embedding of the scheme. This talk is based on joint work with Alexander Gavrilyuk, Jesse Lansdown, and Akihiro Muenemasa.

Contributed talks

Extremal problems for semilinear graphs

Abdul Basit

Monash University, Australia

A very general phenomenon in combinatorial geometry is that geometric graphs exhibit tamer extremal behavior as compared to arbitrary graphs. A semilinear graph is a graph which can be defined using systems of linear equations and inequalities. The notion of semilinear graphs provides a very general framework for studying geometric graphs. We will see some recent extremal results for semilinear graphs. We will also talk about connections of these results to combinatorial geometry and model theory. No background is assumed, and the talk will be accessible to non-experts. Some of these results are joint work with Artem Chernikov, Daniel Horsley, Sergei Starchenko, Terence Tao, and Chieu-Minh Tran.

List-Colouring Graphs with Maximal Local Edge Connectivity 3

Samuel Bastida

Victoria University of Wellington, New Zealand

Brook's Theorem is a fundamental theorem in graph colouring which states that a graph with maximum degree k can be k-coloured except for a small class of exceptions which require $k+1$ colours. In 2018 Stiebitz and Toft obtained a similar result, stating that any graph with maximal local edge connectivity k can be k-coloured except for a small class of exceptions which require $k+1$ colours. List-colouring is an extension of colouring where each vertex is assigned it's own list of k colours. Many theorems about colouring do not extend to list colouring but Brook's Theorem does. We investigate extending the theorem of Stiebitz and Toft to list colouring in the case when $k = 3$.

New discoveries about regular and chiral maps

Marston Conder

University of Auckland, New Zealand

In 2022 Primož Potočnik and I began a programme of extending the list of all orientably-regular maps, from genus 301 up to much larger genus, using some new approaches for finding smooth finite quotients of triangle groups (including the $(2, k, \infty)$ triangle group for certain values of k). Reaching genus 1001 was relatively easy, so then we attempted extending it to genus 1501. This talk will describe the outcome.

Aside from significantly increasing the number of such maps, in both the reflexible and chiral cases, we used the same information to extend the list of all regular *non-orientable* maps. Along the way we developed new means of determining certain information (such as the edge-multiplicity of each map and its dual), and the resulting lists produced some new and in some cases surprising information about various issues, such as the relative proportions of reflexible vs chiral maps, solubility of their automorphism groups, and simplicity of the underlying graphs – resolving an open question in the latter case.

Novel Computational Results for the 3-Longest Paths Problem for Polyhedral Graphs

Max Croucher

University of Canterbury, New Zealand

The 3-Longest Paths Problem is an unsolved problem in Graph Theory. This problem asks whether the three longest paths in a graph share a common vertex. In this presentation, we discuss this problem for 3-connected planar graphs, and present some novel preliminary numerical results. Any counterexample must be non-Hamiltonian, and so we discuss methods for generating and checking these graphs including relevant bounds.

Groups acting on trees and tree-like graphs

Florian Lehner

University of Auckland, New Zealand

Bass-Serre theory is one of the main tools to study groups acting on (infinite) trees. It relates group actions on trees to graphs of groups, describing the groups as iterated amalgamated free products and HNN extensions. Conversely, given a graph of groups, it allows us to construct a group action on a tree. Unfortunately, it is very difficult to use this construction to obtain interesting new examples of groups acting on trees since the actions of vertex and edge stabilisers must satisfy strong compatibility conditions.

To overcome this issue, Reid and Smith recently introduced the theory of local action diagrams. This theory eliminates the need for compatibility conditions, but only allows for the construction

of very specific groups. More precisely, it covers all groups satisfying Tits' property (P), in other words, groups for which there is no interaction between stabilisers of disjoint subtrees.

In this talk we introduce a new ("amalgamated") version of local action diagrams. The resulting theory can be thought of as "in between" Bass-Serre theory and the theory of local action diagrams in the sense that it allows us to model some interaction between stabilisers of disjoint subtrees, at the expense of re-introducing some weak compatibility conditions. (Joint work with C. Lindorfer, R. Möller, and W. Woess)

Exploring spaces of phylogenetic networks

Simone Linz

University of Auckland, New Zealand

Phylogenetic trees and networks are widely used to represent evolutionary relationships between entities such as species, cancer cells, and viruses. A central challenge in phylogenetics is to accurately reconstruct phylogenetic trees and networks from molecular sequence data. The reconstruction process usually necessitates a search through the space of all phylogenetic trees or networks to find an optimal solution. In this talk, we discuss a novel rearrangement operation, called cut edge transfer, to search the space of a particular class of phylogenetic networks that have directed and undirected edges. We show that the space is connected under the operation. Hence, every network in the class can be transformed into any other such network by a sequence of cut edge transfers. This is joint work with Kristina Wicke (New Jersey Institute of Technology).

Intersections of three longest paths in connected graphs

John McLachlan

University of Canterbury, New Zealand

In 1966, Gallai asked the question of whether all longest paths of a connected graph intersect. This question was answered negatively by Walther, and since then, mathematicians have discovered many graphs for which all longest paths do not intersect. In this talk, we will investigate the related conjecture that every three longest paths of a connected graph intersect. We will consider a class of graphs for which this conjecture remains open, and proceed by briefly outlining some known results. In pursuit of proving the conjecture for this class, we will then provide a series of preliminary results, in particular, properties of a theoretical counterexample.

Induced Turán problem

Rajko Nenadov

University of Auckland, New Zealand

Given a bipartite graph H and a graph G in which any pair of large subsets has edge density bounded away from 1, what is the largest number of edges of a subgraph G' of G which does not contain an induced copy of H? We study this Turán-type question and provide tight answer for a large class of bipartite graphs. Joint work with Jacob Fox (Stanford University) and Huy Tuan Pham (Stanford University).

Generalised hexagons and octagons embedded in metasymplectic spaces

Sebastian Petit

University of Canterbury, New Zealand

Generalised polygons play an important role in incidence geometry, building theory and graph theory. A (weak) generalised n-gon can be defined as a point-line geometry such that the incidence graph has diameter n and girth $2n$. In this talk we focus on full embeddings of thick generalised hexagons and octagons in metasymplectic spaces, the geometries related to weak spherical buildings of type F_4. Our main aim is to improve our understanding of how the point relations in the metasymplectic space relate to the possible distances in the embedded generalised polygon. This is based on joint research with Hendrik Van Maldeghem.

Modules of 0-Hecke algebras and quasisymmetric functions in type B

Dominic Searles

University of Otago, New Zealand

We give a uniform combinatorial method for constructing modules of 0-Hecke algebras in all Coxeter types, and apply this to give representation-theoretic meaning to noteworthy families of type B quasisymmetric functions. We construct a type B analogue of Schur Q-functions using domino tableaux, and discuss how these functions correspond to restrictions of certain modules of a type B variant of 0-Hecke–Clifford algebras. This is joint work with Colin Defant.

An equitable partition of the local graphs in the Grassmann graph

Ian Seong

University of Wisconsin-Madison, USA

This talk is about a class of distance-regular graph called the Grassmann graph. Pick vertices x, y of the Grassmann graph such that their distance is strictly between 1 and the diameter. Consider the graph automorphisms that stabilize x and y; we call the group of such automorphisms *the stabilizer of x and y*. The action of this stabilizer on the local graph of x gives rise to an equitable partition. Our goal is to describe this equitable partition.

The positive expansion of quasisymmetric Schur Q-functions into peak Young quasisymmetric Schur functions

Matt Slattery-Holmes

Otago University, New Zealand

The quasisymmetric Schur Q-functions and the peak Young quasisymmetric Schur functions are analogues of the famous Schur functions, and form additive bases of the peak subalgebra of quasisymmetric functions. We provide a combinatorial formula, in terms of tableau diagrams, for the (necessarily non-negative) coefficients when one expands a quasisymmetric Schur Q-function into peak Young quasisymmetric Schur functions

Remarks on the Klein types of finite Minkowski planes

Günter Steinke

University of Canterbury, New Zealand

Minkowski planes are incidence geometries with points, lines (also called generators) and blocks (normally called circles). They are extensions of affine planes by a family of hyperbolic ovals. A finite Minkowski plane of order n is equivalent to a sharply 3-transitive set of permutations of degree $n+1$. All known finite Minkowski planes have order a prime power q and correspond to $\mathsf{PSL}(2,q) \cup (\mathsf{PGL}(2,q) \setminus \mathsf{PSL}(2,q))\alpha$ where α is an automorphism of the Galois field of order q. In case of α being the identity one obtains the so-called miquelian Minkowski planes. These are realised as the geometry of plane sections of a ruled quadric in 3-dimensional projective space. Furthermore, all finite Minkowski planes of even order are miquelian.

Similar to the Lenz-Barlotti classification of projective planes Monica Klein classified in 1992 the set of all 2-sets $\{p, p'\}$ for which the group of all $\{p, p'\}$-homotheties in a given group Γ of automorphisms of a Minkowski plane $\mathcal{M}$ is transitive on $C \setminus \{p, p'\}$ where C is any circle through p and p' and where a $\{p, p'\}$-*homothety* is an automorphism of $\mathcal{M}$ that fixes the two points p and p' and every circle through p and p'. For many of the 23 possible configurations (the so-called Klein types of Γ) it is known that they can only occur for proper subgroups of the full automorphism group of finite miquelian Minkowski planes.

In this talk I review what is known about the Klein types of finite Minkowski planes, that is, the types of the full automorphism groups of these planes with respect to homotheties. I further discuss some "large" types where the automorphism groups of the planes are sufficiently large that may make it possible to reconstruct the underlying Minkowski planes.

Soluble regular hypermaps

Darius Young

University of Auckland, New Zealand

Using a theorem of Singerman to categorise derived groups of ordinary triangle groups. Leading to a proof that for any type (r, s, t) with r, s, t not pairwise coprime, there exist regular hypermaps of that type with derived length 3 or less, and furthermore regular hypermaps of that type with any greater derived length.

Participants

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